

Hermitian D-brane solutions

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Abstract

A low-energy background field solution describing D-membrane configurations is constructed which is distinguished by the appearance of a Hermitian metric on the internal space. This metric is composed of a number of independent harmonic functions on the transverse space. Thus this construction generalizes the usual harmonic superposition rule. The BPS bound of these solutions is shown to be saturated indicating that they are supersymmetric. By means of T-duality, we construct more solutions of the IIA and IIB theories.

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1 Introduction

In the past two years, evidence has accumulated which indicates that all five superstring theories are in fact perturbative expansions about different points in the phase space of some more fundamental theory, the so-called M-theory [1]. In particular, this interpretation is suggested by the success in relating these theories by various duality transformations. This progress has been marked by the realization that extended objects, other than just strings, play a very important role. Of note in the type IIA and IIB superstring theories are Dirichlet branes (D-branes) which have been recognized as the carriers of Ramond-Ramond (RR) charges [2] – see also [3].

One of the remarkable aspects of D-branes is that they can be analysed as external sources in the framework of perturbative worldsheet calculations. When the effective coupling between the strings and D-branes grows, a complementary description of the D-branes is provided by solutions of the effective low energy supergravity equations of motion. Investigating both of these approaches to D-brane physics has proven useful. In particular, both methods converged in recent calculations of black hole entropy [4]. In this analysis, perturbative techniques were used in making a counting of the ground state degeneracy of certain D-brane bound states, while the corresponding low energy solutions provided a black hole background for which one could calculate the usual Hawking-Bekenstein entropy. The precise matching of the results in these two regimes has provided some striking new insights into the underlying microscopic degrees of freedom for (at least near-extremal) black holes.

One aspect of D-brane physics which has recently attracted some attention [5, 6, 7, 8, 9] is the possibility of constructing marginally bound states in which the D-branes intersect at angles [10], other than 0 and $\pi/2$. In these configurations, the various D-branes are typically related by an $SU(N)$ subgroup of rotations, in order to preserve supersymmetry — actually [5, 11] indicates that other subgroups are possible. An explicit background field solution describing several D-membranes rotated by $SU(2)$ rotations was presented in [7]. The general form of this solution was complicated and did not appear simply related to any previously known solutions.

In this paper, we will show that this solution with angled D-membranes falls within a class of D-brane solutions characterized by a Hermitian metric on the internal space. This metric contains a number of independent harmonic functions, and so the new solution represents a generalization of the usual harmonic superposition construction [12]. These solutions, presented in section 2, describe a general class of six-dimensional D-membrane configurations. In section 3, we provide a similar class of four-dimensional solutions describing configurations of D-membranes, as well as D6-branes, wrapped around the internal space¹. For these solutions, the metric on the internal six-torus is again Hermitian. We also explicitly show that this solution saturates the BPS bound, and preserves one-eighth of the supersymmetries. The following section is devoted to building some simple extensions

¹While this work was being completed, a paper by Balasubramanian, Larsen and Leigh [13] appeared which describes this solution in terms of D-membranes at angles.

of this Hermitian metric solution through T-duality transformations. We conclude with a brief discussion of the results in section 5. Our notation and conventions follow those established in [14].

2 Six-dimensional solutions

Ref. [7] presented a solution describing an arbitrary number of D-membranes oriented at angles with respect to one another. There, this solution was derived directly by solving the low-energy supergravity equations of motion — see also [8]. Later, the two membrane solution was also reproduced through a series of boosts and duality transformations applied to two orthogonal M-branes [9]. As presented in [7], the solution took a complicated form in terms of harmonic functions X_a and rotation angles α_a , where the index a is labeling the a 'th D-membrane. In particular following the analysis of ref. [10], the membrane rotations were described as a particular set of $SU(2)$ transformations, however no use of complex coordinates was made in presenting the solution. One finds however that the latter can be employed to greatly simplify, as well as generalize, this solution.

We begin by defining complex coordinates on the effective worldvolume²:

$$z^1 = \frac{1}{\sqrt{2}} (y^1 + iy^2) \quad \text{and} \quad z^2 = \frac{1}{\sqrt{2}} (y^3 + iy^4) \quad (1)$$

together with their complex conjugates, \bar{z}^1 and \bar{z}^2 . We then have the solution

$$\begin{aligned} ds^2 &= \sqrt{\mathcal{H}} \left(\frac{-dt^2 + 2 H_{a\bar{b}} dz^a d\bar{z}^b}{\mathcal{H}} + \sum_{i=5}^9 (dx^i)^2 \right) \\ A^{(3)} &= \pm \frac{i}{\mathcal{H}} H_{a\bar{b}} dt \wedge dz^a \wedge d\bar{z}^b \\ e^{2\phi} &= \mathcal{H}^{\frac{1}{2}} \end{aligned} \quad (2)$$

where $H_{a\bar{b}}$ is a Hermitian matrix given by

$$H_{a\bar{b}} = \begin{pmatrix} 1 + A & C \\ C^* & 1 + B \end{pmatrix} \quad (3)$$

and $\mathcal{H}^2 = \det H = (1 + A)(1 + B) - |C|^2$. The entries of $H_{a\bar{b}}$ are independent harmonic functions of the x^i 's. That is, they solve the flat space Poisson's equation in the transverse space, *e.g.*, $\delta^{ij} \partial_i \partial_j A = \text{sources}$. Implicitly here, we assume that these harmonic functions all vanish asymptotically leaving $H_{a\bar{b}} \simeq \delta_{a\bar{b}}$. At the same time, we think of the y^i coordinates as being compact with range $2\pi L_i$, and hence asymptotically the internal space is

²We include any directions in which the D-branes are delocalized as world-volume coordinates. We will refer to all of the spatial (effective) world-volume coordinates with y^i , while the transverse coordinates will be x^i .

simply a square four-torus. One can easily change the asymptotic moduli of this internal space by either modifying these periodicities or by adding constants to the harmonic functions in eq. (3). One must demand, however, that $H_{a\bar{b}}$ remains invertible. We also note that the combination

$$\det(H) H^{\bar{a}b} = \begin{pmatrix} 1+B & -C \\ -C^* & 1+A \end{pmatrix} \quad (4)$$

takes a simple form in terms of the harmonic functions. Here, $H^{\bar{a}b}$ denotes the inverse of the Hermitian metric.

If we choose zero for the (complex) off-diagonal component of $H_{a\bar{b}}$, *i.e.*, $C = 0$, then the above solution simply reduces to that describing orthogonal D-membranes, constructed with the usual harmonic superposition rule [12]. To relate the above solution to that of angled D-membranes [7], one makes the following choice for the harmonic functions³:

$$\begin{aligned} A &= \sum_{a=1}^n X_a \cos^2 \alpha_a \\ B &= \sum_{a=1}^n X_a \sin^2 \alpha_a \\ C = C^* &= \sum_{a=1}^n X_a \sin \alpha_a \cos \alpha_a \end{aligned}$$

In this case, the rotations acting on the individual membranes can be described as SU(2) transformations: $z^b \rightarrow [U_a]^b{}_c z^c = [\exp(i\alpha_a \sigma_2)]^b{}_c z^c$. Our Hermitian metric can then be written as

$$H_{b\bar{c}} = \delta_{b\bar{c}} + \sum_{a=1}^n X_a [U_a]^1{}_b [U_a^*]^1{}_{\bar{c}} \quad (5)$$

Written in this form, we can easily generalize the solution by allowing the matrices U_a to be arbitrary SU(2) transformations.

Following [7], it is straightforward to calculate the ADM mass and charge densities for the general solution, and further to show that these saturate the BPS bound. Hence these solutions preserve one-quarter of the supersymmetries. We leave the details of these calculations for the next section, where the four-dimensional solution is discussed.

It is interesting to consider the special case where all of the harmonic functions have a common centers⁴. With a single center, the Hermitian metric takes the form

$$H_{a\bar{b}} = \delta_{a\bar{b}} + \frac{h_{a\bar{b}}}{|\vec{x} - \vec{x}_o|^3} \quad (6)$$

³Actually this only reproduces the solution of [7] up to an interchange of the internal coordinates, $y^2 \leftrightarrow y^3$, and a gauge transformation of the RR potential.

⁴We are grateful to A.A. Tseytlin for interesting discussions on this point.

where $h_{a\bar{b}}$ is a *constant* Hermitian matrix. In this case, however, it will always be possible to find an $SU(2)$ transformation which diagonalizes the latter, *e.g.*,

$$\tilde{h} = U h U^\dagger = \begin{pmatrix} \tilde{h}_{11} & 0 \\ 0 & \tilde{h}_{22} \end{pmatrix} . \quad (7)$$

Therefore by performing the (global) coordinate transformation $\tilde{z}^i = U^i_j z^j$, the off-diagonal components of $H_{a\bar{b}}$ will be eliminated in which case the single-center solution is reduced to that describing orthogonal D-membranes [12]. Thus in the special case where all of the centers for the harmonic functions come together, our new Hermitian D-membrane solutions in fact only reproduce the known orthogonal membrane solution. These considerations may be generalized to the special case of several common centers

$$H_{a\bar{b}} = \delta_{a\bar{b}} + \sum_{i=1}^n \frac{h_{a\bar{b}}^{(i)}}{|\vec{x} - \vec{x}_i|^3} \quad (8)$$

where $h_{a\bar{b}}^{(i)} = \mu_i h_{a\bar{b}}^{(1)} + \rho_i \delta_{a\bar{b}}$ with constants μ_i and ρ_i for $i \geq 2$. Again, a coordinate transformation can be found which will reduce this special case of (2) to the standard orthogonal solution. Thus while generically our Hermitian solutions represent new solutions, at many points in the moduli space they reduce to known solutions [12]. Further, however, irrespective of the choice of the harmonic functions in $H_{a\bar{b}}$, as long as the centers all have finite separation then the asymptotic fields will take the form given in eq. (6). Therefore at long distances, physicists may always interpret these general D-membrane configurations in terms of orthogonal membranes.

3 Four-dimensional solutions

In this section, we will generalize the above solutions to the case where the effective world-volume or the internal space occupies six spatial dimensions leaving four noncompact directions. Thus the D-membranes fill a six-torus described by y^i where $i = 1, 2, 3, 4, 5, 6$, while there are three transverse spatial coordinates, x^i with $i = 7, 8, 9$. Since parallel D2- and D6-branes are supersymmetric [3], it is natural to extend these solutions to include D6-branes wrapped around the six-torus. Again, complex coordinates are useful and we define

$$z^a = \frac{1}{\sqrt{2}} (y^{2a-1} + i y^{2a}) \quad (9)$$

for $a = 1, 2, 3$. The natural generalization of eq. (2) is then⁵

$$ds^2 = \sqrt{\mathcal{H}_2 \mathcal{H}_6} \left(\frac{-dt^2 + 2 H_{a\bar{b}} dz^a d\bar{z}^b}{\mathcal{H}_2 \mathcal{H}_6} + \sum_{i=7}^9 (dx^i)^2 \right)$$

⁵We have introduced an electric seven-form potential to describe D6-branes. This represents a great simplification over the usual magnetic one-form RR potential in describing multi-center solutions. We also note here that the solutions here and in the previous section are given in terms of the string-frame metric.

$$\begin{aligned}
A^{(3)} &= \pm \frac{i}{\mathcal{H}_2} H_{a\bar{b}} dt \wedge dz^a \wedge d\bar{z}^b \\
A^{(7)} &= \pm \frac{i}{\mathcal{H}_6} dt \wedge dz^1 \wedge dz^2 \wedge dz^3 \wedge d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3 \\
e^{2\phi} &= \frac{\mathcal{H}_2^{1/2}}{\mathcal{H}_6^{3/2}}
\end{aligned} \tag{10}$$

The Hermitian matrix $H_{a\bar{b}}$ takes a slightly more complicated form

$$H_{a\bar{b}} = \begin{pmatrix} (1+A)(1+D) - |E|^2 & C(1+D) + E^*F^* & F^*(1+A) + CE \\ C^*(1+D) + EF & (1+B)(1+D) - |F|^2 & E(1+B) + C^*F^* \\ F(1+A) + C^*E^* & E^*(1+B) + CF & (1+A)(1+B) - |C|^2 \end{pmatrix}. \tag{11}$$

and in this case

$$\begin{aligned}
\mathcal{H}_2 &= \sqrt{\det H} \\
&= (1+A)(1+B)(1+D) - |C|^2(1+D) - |F|^2(1+A) - |E|^2(1+B) \\
&\quad - CEF - C^*E^*F^*
\end{aligned} \tag{12}$$

In this case, however, the inverse matrix (combined with the determinant) still takes a simple form

$$\sqrt{\det H} H^{\bar{a}b} = \begin{pmatrix} 1+B & -C & -F^* \\ -C^* & 1+A & -E \\ -F & -E^* & 1+D \end{pmatrix}. \tag{13}$$

For the D6-branes, we have $\mathcal{H}_6 = 1+G$. Now the above fields satisfy the low energy supergravity equations of motion if A, B, \dots, G are again harmonic functions on the transverse space. Assuming that these harmonic functions all vanish asymptotically, we would have

$$A = \sum_{i=1}^n \frac{a_i}{|\vec{x} - \vec{x}_i|} \tag{14}$$

and similarly for the other harmonic functions. From eq. (13), one easily sees that by setting $D = E = F = 0$, one would recover the D-membrane configuration of the previous section. However, the membranes would now be delocalized in y^5 and y^6 .

As in the six-dimensional solution, removing the off-diagonal components of $H_{a\bar{b}}$, by choosing $C = E = F = 0$, reproduces the standard solution describing a configuration of orthogonal D-membranes and parallel D6-branes [12]. In fact as before, in the special case that all of the harmonic functions have a single common center, our Hermitian metric solution can be reduced to this known form by a coordinate transformation on the world-volume coordinates, z^a . For generic harmonic functions, however, our solution represents a generalization of the usual harmonic superposition construction of extremal D-brane solutions [12]. One may extend the form (5) to the present 3×3 Hermitian matrix,

$$H_{a\bar{b}} = \delta_{a\bar{b}} + \sum_{i=1}^n X_i [U_i]^1{}_a [U_i^*]^1{}_{\bar{b}} \tag{15}$$

where the X_i are harmonic functions on the transverse space, and the U_i are now $SU(3)$ transformations acting on $z^{1,2,3}$. With these choices, the Hermitian metric solution manifestly describes D-membranes oriented at angles.

3.1 Supersymmetry

In this section, we show that the solution (10) saturates the BPS bound, and hence that it preserves one-eighth of the supersymmetries. In this calculation, we begin by computing the ADM mass and the RR charge densities, using the appropriate asymptotic flux integrals. These densities are then completely determined by the leading-order asymptotic behavior of the harmonic functions, which we define by

$$\frac{1}{\sqrt{\det H}} H_{a\bar{b}} = \delta_{a\bar{b}} - \frac{h_{a\bar{b}}}{r} \quad (16)$$

where $h_{a\bar{b}}$ is a constant Hermitian matrix. Also we set $G = g/r$, and we are using $r^2 = (x^7)^2 + (x^8)^2 + (x^9)^2$. Next, we examine the Bogomol'nyi matrices of the individual constituents in order to prove that the solution (10) preserves certain supersymmetries.

The ADM mass per unit six-volume is defined by [15]:

$$m = \frac{1}{2\kappa^2} \oint \sum_{i=7}^9 R^i \left[\sum_{j=7}^9 \partial_j \delta g_{ij} - \sum_{j=1}^9 \partial_i \delta g_{jj} \right] r^2 d^2 \Omega \quad (17)$$

where R^i is a radial unit vector in the transverse space, and δg is the deviation of the asymptotic Einstein-frame metric g^E from flat space⁶:

$$\delta g_{\mu\nu} = g_{\mu\nu}^E - \eta_{\mu\nu} . \quad (18)$$

Using the asymptotic fields (16) as described above, the ADM mass density becomes

$$m = \frac{2\pi}{\kappa^2} (\text{Tr}[h] + g). \quad (19)$$

We note that this result is independent of the three (complex) constants appearing in the off-diagonal components of $h_{a\bar{b}}$

The D-membranes produce asymptotic electric fields in the RR four-form *i.e.*, $F_{tr a\bar{b}}^{(4)} = -\partial_r A_{ta\bar{b}}^{(3)}$. If we define (asymptotically orthonormal) basis vectors, $n^a = \partial_{z^a}$ and their complex conjugates, then we may define the associated set of electric charge densities as⁷

$$\tilde{q}_{a\bar{b}} = \frac{1}{\sqrt{2}\kappa} \oint *^{(4)} [i_{n^a} i_{\bar{n}^b} F^{(4)}] \quad (20)$$

⁶The Einstein-frame metric is related to the string-frame metric, which appears in the solutions, by $g_{\mu\nu}^E = e^{-\frac{\phi}{2}} g_{\mu\nu}$.

⁷Note that this definition is equivalent to that in [7]. There, the components of $\tilde{q}_{a\bar{b}}$ would be calculated by taking the ten-dimensional Hodge dual of $F^{(4)}$, integrating the resulting fluxes over the asymptotic two-sphere and four directions in the six-torus, but then dividing out by the volume of the latter internal subspace.

where $\ast^{(4)}$ denotes Hodge duality on the noncompact four-dimensional space, and i_v denotes the usual interior product of vectors with forms. The integral is performed over the asymptotic two-sphere in the transverse space. The simple final result is

$$\tilde{q}_{a\bar{b}} = \pm \frac{2\sqrt{2}\pi i}{\kappa} h_{a\bar{b}} \quad (21)$$

Note that since $i_{n^a} i_{n^b} F^{(4)} = 0$, there are no fluxes or charges associated with the (2,0) or (0,2) cycles of the internal six-torus.

The D6-branes provide a magnetic RR charge defined by:

$$p = \frac{1}{\sqrt{2}\kappa} \oint F^{(2)} = \frac{1}{\sqrt{2}\kappa} \oint \ast F^{(8)} \quad (22)$$

where $F^{(8)} = dA^{(7)}$ is the field strength of the 7-form potential of the solution (10), and again the integral is performed over the asymptotic two-sphere. The resulting charge density is

$$p = \pm \frac{2\sqrt{2}\pi}{\kappa} g \quad (23)$$

To investigate the supersymmetries, we examine the Bogomol'nyi matrix [16, 18], which is derived using both the supersymmetry algebra and the asymptotic form of the background fields. The BPS bound is saturated and supersymmetries are preserved when this matrix has eigenspinors with vanishing eigenvalue. Following our conventions⁸, one finds that the Bogomol'nyi matrix \mathcal{M} is given by [18]:

$$\mathcal{M} = m \mathbb{1} + \frac{i}{\sqrt{2}\kappa} \tilde{q}_{a\bar{b}} \Gamma_0 \Gamma_{[a} \Gamma_{\bar{b}]} - \frac{i}{\sqrt{2}\kappa} p \Gamma_7 \Gamma_8 \Gamma_9 \Gamma_{10} \quad (24)$$

where $\mathbb{1}$ denotes the identity matrix, and $[]$ is used to indicate antisymmetrization, as in $\Gamma_{[a} \Gamma_{\bar{b}]} = (\Gamma_a \Gamma_{\bar{b}} - \Gamma_{\bar{b}} \Gamma_a)/2$. Now unbroken supersymmetries will be signaled by the presence of eigenspinors satisfying $\mathcal{M}\varepsilon = 0$.

A more insightful approach is to consider pulling apart the individual constituents of the general solution, *i.e.*, arrange the centers of the independent harmonic functions far apart, and then examine the individual constraints that arise from each component. Hence we begin by considering the solution (10) for which $g = 0$ and only $h_{1\bar{1}}$ is nonvanishing in eq. (16). This solution would represent an isolated D-membrane lying in the (y^1, y^2) plane (and also delocalized in the remaining internal directions).

$$\mathcal{M} = \frac{2\pi}{\kappa^2} h_{1\bar{1}} (\mathbb{1} \pm i \Gamma_0 \Gamma_{y^1} \Gamma_{y^2}) \quad (25)$$

⁸Our ten-dimensional Dirac matrices satisfy $\{\Gamma_\mu, \Gamma_\nu\} = -2g_{\mu\nu}$, and $\Gamma_{10} = \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 \Gamma_6 \Gamma_7 \Gamma_8 \Gamma_9$ so that $(\Gamma_{10})^2 = +\mathbb{1}$. Also note that for the complex indices, we use $\Gamma_a = (\Gamma_{y^{2a-1}} + i\Gamma_{y^{2a}})/\sqrt{2}$ and $\Gamma_{\bar{a}} = (\Gamma_{y^{2a-1}} - i\Gamma_{y^{2a}})/\sqrt{2}$.

The matrix enclosed in the brackets has precisely two distinct eigenvalues, 0 and 2. The eigenspinors with vanishing eigenvalue are those satisfying

$$\varepsilon_+ = \mp i \Gamma_0 \Gamma_{y^1} \Gamma_{y^2} \varepsilon_- \quad (26)$$

where we have separated the two chiral components ε_{\pm} defined by:

$$\varepsilon_{\pm} = \frac{1}{2} (\mathbb{1} \pm \Gamma_{10}) \varepsilon . \quad (27)$$

Thus we have recovered the well-known result that half of the supersymmetries are preserved by a D-membrane, and the eigenspinors satisfy precisely the constraint (26) arising from the superstring worldsheet analysis of D-branes [3]. Similarly, the other diagonal cases where only $h_{2\bar{2}} \neq 0$ or $h_{3\bar{3}} \neq 0$ lead to

$$\varepsilon_+ = \mp i \Gamma_0 \Gamma_{y^3} \Gamma_{y^4} \varepsilon_- \quad (28)$$

$$\varepsilon_+ = \mp i \Gamma_0 \Gamma_{y^5} \Gamma_{y^6} \varepsilon_- \quad (29)$$

respectively, as expected for a D-membrane lying in the (y^3, y^4) or the (y^5, y^6) plane. There are common solutions to these constraints (26-29) leaving one-eighth of the supersymmetries unbroken [12].

The analysis of the case where only $g \neq 0$, representing an isolated D6-brane, is also straightforward. In this case the Bogomol'nyi matrix is given by

$$\begin{aligned} \mathcal{M} &= \frac{2\pi}{\kappa^2} g (\mathbb{1} \mp i \Gamma_{x^7} \Gamma_{x^8} \Gamma_{x^9} \Gamma_{10}) \\ &= \frac{2\pi}{\kappa^2} g (\mathbb{1} \pm i \Gamma_0 \Gamma_{y^1} \Gamma_{y^2} \Gamma_{y^3} \Gamma_{y^4} \Gamma_{y^5} \Gamma_{y^6}) . \end{aligned} \quad (30)$$

Here eigenspinors with vanishing eigenvalue satisfy the expected D6-brane constraint:

$$\varepsilon_+ = \mp i \Gamma_0 \Gamma_{y^1} \Gamma_{y^2} \Gamma_{y^3} \Gamma_{y^4} \Gamma_{y^5} \Gamma_{y^6} \varepsilon_- . \quad (31)$$

It is also straightforward to show that the common eigenspinors satisfying (26-29) also satisfy the above constraint.

We must also consider some other configurations in which only off-diagonal components of $h_{a\bar{b}}$ are nonvanishing. These are unusual since from eq. (19) one finds that $m = 0$ for these cases — we will comment more on these configurations in section 5. Consider the solution with only $h_{1\bar{2}} = h_{2\bar{1}} \neq 0$ in which case the Bogomol'nyi matrix reduces to

$$\mathcal{M} = \mp i \frac{2\pi}{\kappa^2} h_{1\bar{2}} \Gamma_0 (\Gamma_{y^1} \Gamma_{y^4} + \Gamma_{y^2} \Gamma_{y^3}) . \quad (32)$$

The appropriate eigenspinors must satisfy

$$\Gamma_{y^1} \Gamma_{y^4} \varepsilon_{\pm} = -\Gamma_{y^2} \Gamma_{y^3} \varepsilon_{\pm} \quad (33)$$

and it is easy to show that the common eigenspinors of eqs. (26) and (28) will satisfy this constraint. Similarly the new constraints that are produced by considering the other off-diagonal components of $h_{a\bar{b}}$ are already satisfied by the common eigenspinors satisfying eqs. (26-29). Therefore these contributions do not lead to the breaking of further supersymmetries. Hence we arrive at the final conclusion that the four-dimensional Hermitian solution (10) preserves one-eighth of the supersymmetries.

4 T-dual solutions

The ten-dimensional T-duality transformations between the type IIA and IIB superstring theories may be found in ref. [17] — see also [14]. These transformations provide a powerful tool for generating new solutions from a given background field configuration. An important feature of T-duality is that it respects supersymmetry. Therefore, all T-dual solutions of (10) will also preserve one-eighth of the supersymmetries. In general, the T-dual solutions will involve various non-threshold D-brane bound states (as in [14]), but we will restrict ourselves to two cases which have a simple interpretation as a marginally bound system of one or two kinds of D-branes.

4.1 4-4-4-0 configuration

A particularly simple solution is obtained by T-dualizing all of the six-torus coordinates in (10). The resulting background field solution describes a system of D4- and D0-branes. When the entire toroidally compactified space is T-dualized, the sum of the internal metric with the Kalb-Ramond field is replaced by its inverse [19]: $g + B \rightarrow (g + B)^{-1}$. In the present case without a Kalb-Ramond field then, the internal metric is transformed to its inverse and so in the final solution, the internal metric is again Hermitian. The background fields of the T-dual solution may be written⁹:

$$\begin{aligned}
ds^2 &= \sqrt{\mathcal{H}_0 \mathcal{H}_4} \left(-\frac{dt^2}{\mathcal{H}_0 \mathcal{H}_4} + 2 \frac{\widetilde{H}_{a\bar{b}} dz^a d\bar{z}^b}{\mathcal{H}_4} + \sum_{i=7}^9 (dx^i)^2 \right) \\
A^{(5)} &= \pm \widetilde{H}^{\bar{a}a} \epsilon_{\bar{a}\bar{b}\bar{c}} \epsilon_{abc} dt \wedge dz^b \wedge dz^c \wedge d\bar{z}^{\bar{b}} \wedge d\bar{z}^{\bar{c}} \\
A^{(1)} &= \mp \frac{1}{\mathcal{H}_0} dt \\
e^{2\phi} &= \frac{\mathcal{H}_0^{3/2}}{\mathcal{H}_4^{1/2}}
\end{aligned} \tag{34}$$

where ϵ_{abc} is the three dimensional Levi-Civita symbol. The components of the new hermitian matrix \widetilde{H} are related to the inverse of H as

$$\widetilde{H}_{a\bar{b}} = \sqrt{\det H} H^{\bar{a}b} . \tag{35}$$

⁹Here, rather than writing the RR field of the D4-branes as a magnetic three-form potential, we use an electric five-form potential.

We have also introduced the notation: $\mathcal{H}_0 = 1 + G = \mathcal{H}_6$, and $\mathcal{H}_4 = \det(\widetilde{H}) = \sqrt{\det H} = \mathcal{H}_2$, where the latter is explicitly given in eq. (12). If the off-diagonal components of \widetilde{H} are set to zero, the above solution reduces to a marginally bound system of orthogonal D4-branes and D0-branes, which was considered by [20] in investigating black hole entropy. We also remark that if this approach of T-dualizing the entire compact space were applied to the six-dimensional solutions of section 2, the final result would again be in the same family of D-membrane solutions presented there.

4.2 3-3-3-3 configuration

An interesting solution of the Type IIB theory is generated by T-dualizing only half of the internal compact space. If we restrict the hermitian matrix H in eq. (10) to be real, one can construct a solution composed entirely of D3-branes. Hence after making this restriction, we dualize on the internal coordinates $\{y^2, y^4, y^6\}$ to obtain

$$\begin{aligned}
ds^2 &= \sqrt{\mathcal{H}\mathcal{H}'} \left(\frac{-dt^2 + H_{ab} dy^{2a-1} dy^{2b-1}}{\mathcal{H}\mathcal{H}'} + \frac{\widetilde{H}_{ab} dy^{2a} dy^{2b}}{\mathcal{H}} + \sum_{i=7}^9 (dx^i)^2 \right) \\
F^{(5)} &= \mp \frac{\partial_i \mathcal{H}'}{\mathcal{H}'^2} dt \wedge dy^1 \wedge dy^3 \wedge dy^5 \wedge dx^i \pm \epsilon_{ijk} \partial_i \mathcal{H}' dy^2 \wedge dy^4 \wedge dy^6 \wedge dx^j \wedge dx^k \\
&\quad \pm \partial_i \left(\frac{H_{ab}}{\mathcal{H}} \right) \epsilon_{bcd} dt \wedge dy^{2a-1} \wedge dy^{2c} \wedge dy^{2d} \wedge dx^i \\
&\quad \pm \epsilon_{ijk} \partial_i (\mathcal{H} H^{ab}) \epsilon_{bcd} dy^{2a} \wedge dy^{2c-1} \wedge dy^{2d-1} \wedge dx^j \wedge dx^k . \\
e^{2\phi} &= 1
\end{aligned} \tag{36}$$

Here, the (real symmetric) matrices, H_{ab} and \widetilde{H}_{ab} , are defined by eqs. (11) and (35), respectively, with the restrictions that $C = C^*$, $E = E^*$ and $F = F^*$. We also denote $\mathcal{H} = \det \widetilde{H} = \sqrt{\det H} = \mathcal{H}_2$ and $\mathcal{H}' = 1 + G = \mathcal{H}_6$. If we set the off-diagonal components to zero, *i.e.*, $C = 0 = E = F$, then the solution describes an orthogonal system of D3-branes oriented in the (y^1, y^3, y^5) , (y^1, y^4, y^6) , (y^2, y^3, y^6) and (y^2, y^4, y^5) surfaces. These solutions were first considered in refs. [20, 21].

5 Discussion

In this paper, we have presented low energy background field solutions describing D-membrane configurations which are characterized by a Hermitian metric on the effective worldvolume directions. The components of this metric are composed of independent harmonic functions on the transverse space. Thus these solutions generalize the usual harmonic superposition rule for the construction of D-brane bound states [12]. We have explicitly shown that these new configurations saturate the BPS bound, preserving one-eighth (one-quarter) of the supersymmetries in four (six) dimensions. The six-dimensional solution of section 2 generalizes the solution describing D-membranes at angles presented in [7]. In

the four-dimensional solution, the harmonic functions may be chosen so that the solution manifestly describes D-membranes oriented at $SU(3)$ angles with respect to one another. The latter is in accord with the analysis of [10]. Their worldsheet techniques can be used to show that the supersymmetry restrictions imposed by D-membranes at $SU(3)$ angles are compatible, leaving one-eighth of the supersymmetries unbroken. In fact, the latter analysis is essentially the same as that of the Bogomol'nyi matrix in section 3.1.

Generically, the Hermitian metric solutions generalize the standard harmonic superposition rule [12]. When the off-diagonal components of the metric vanish, it is easily seen that our solutions reduce to the orthogonal D-membrane solutions constructed by the latter methods. Further, however, at special points in the moduli space — in particular, when all of the harmonic functions have a single common center — the new solutions may be reduced to the known orthogonal D-membrane solutions by a unitary transformation of the internal coordinates. Since physical quantities like the ADM mass and Ramond-Ramond charge densities do not depend on the short-range structure of the solution, even a multi-center solution will appear to have the same properties as some single center solution, and hence some system of orthogonal D-membranes. Hence the new solutions and the standard orthogonal configurations would be indistinguishable for a far field observer. Of course, the difference between the orthogonal and the general Hermitian solutions will be significant when the short-range physics is important.

The solutions constructed here and by the usual harmonic superposition rule are completely delocalized in the internal directions. Thus even though these solutions may have an interpretation in terms of, *e.g.*, orthogonal D-membranes, they show no structure in the directions parallel to the worldvolume of one or more of the D-branes. Finding solutions which explicitly display all of the structure of such intersecting D-branes is a difficult and unresolved problem — see [22, 5, 11]. The possibility to separate the centers of the harmonic functions does provide some insight into the short distance structure of these configurations. For a given single center solution describing a configuration of intersecting D-membranes, one can see from the preceding discussion that there is not a unique way to resolve the short distance fields. In particular, while one can separate the centers in accord with the standard orthogonal solution, the Hermitian metric solution can provide a distinct resolution of the short range fields in terms of D-membranes oriented at angles. Thus in smearing out or delocalizing the solutions, one has in fact lost more information about the short distance physics than might have been previously expected.

An interesting question is whether or not this non-uniqueness has any implications for the black hole entropy calculations performed in a D-brane framework. In fact, if one focuses on the single center solutions the inclusion of D6-branes along with the D-membranes in the four-dimensional solutions of section 3 generically produces a nonsingular black hole solution. Must one then take into account all possible orientations of D-membranes which might describe a black hole with a given set of four-dimensional charges in calculating the ground state degeneracy? We expect that the answer is no, to leading order. While the above discussion indicates that here and perhaps in other cases, there may be more configurations than considered at first sight, this will be a subleading contribution, which

does not affect the dominant exponential degeneracy. It would be rather like considering contributions of the D-string winding states in the standard five-dimensional calculations [4]. In principle, all of these configurations make distinct contributions in any regime of interest, however, it is sufficient to focus on a single winding configuration to determine the leading order degeneracy [4, 23]. Still the non-uniqueness discovered here would be important if one wishes to make a detailed comparison of subleading contributions to the black hole entropy.

In examining the Bogomol'nyi matrices in section 3.1, some interesting configurations were considered for which the mass vanished. For concreteness, let us consider the six-dimensional solution (2) with $A = 0 = B$ and $C = C^* = c/|\vec{x}|^3$. It is straightforward to show that this solution has a vanishing mass, but nonvanishing charges. The solution is singular at $C = 1$ where $\mathcal{H} = 0$ — the presence of the singularity can be determined by examining the curvature, or more easily by considering simpler invariants such as $(\nabla\phi)^2$. At this point, the Hermitian metric is degenerate as the y^1 and y^3 directions, as well as y^2 and y^4 , become collinear. Note however that in the string frame metric, the proper volume of the internal four-torus remains finite as $C \rightarrow 1$ because of the factors of $\mathcal{H}^{-1/2}$ in $G_{\bar{a}\bar{b}}$. Since this example is a single center solution, one can consider which orthogonal solution results upon making a coordinate transformation (7). In this case, the transformed diagonal metric has the form

$$H = \begin{pmatrix} 1 + \frac{c}{|\vec{x}|^3} & 0 \\ 0 & 1 - \frac{c}{|\vec{x}|^3} \end{pmatrix} . \quad (37)$$

Hence the corresponding configuration of two orthogonal D-membranes has a vanishing mass because the mass density of one of the constituents is actually negative! This unsavory feature may indicate that these solutions should not be regarded as physical. Finally, we note that these curious configurations are reminiscent of massless black hole solutions found in the low energy heterotic string theory [24].

It would be of interest to more fully examine the family of new solutions which might be generated from those presented here by means of T-duality. Raising these Type IIA supergravity solutions to eleven-dimensional solutions would also be an interesting avenue of exploration. Finally we note that there would be no new (stationary) nonextremal solutions. In a nonextremal configuration, the branes would collapse forming a black hole at a particular position. Thus the final configuration would be described by some single center solution. However, one should expect that as for the extremal solutions these nonextremal single center solutions will be equivalent up to a coordinate transformation to the standard solutions [25], which are interpreted in terms of orthogonal branes.

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